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Bounded confidence and social networks

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Abstract. In the so-called bounded confidence model proposed by Deffuant et al, agents can influence each other's opinion provided that the opinions are already sufficiently close enough. We discuss here the influence of possible social network topologies on the dynamics of this model.

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1 Introduction

Many models about opinion dynamics, [1–3], are based on binary opinions which social actors update as a result of social influence, often according to some version of a majority rule. Binary opinion dynamics have been well studied, such as the herd behaviour described by economists [1,3,4]. When binary interactions involve any pair of randomly chosen agents, the attractors of the dynamics display a uniformity of opinions, either 0 or 1. Clusters of opposite opinions appear when the dynamics occur on a social network with exchanges restricted to connected agents. These patterns resemble magnetic domains in Ising ferromagnets.

The spreading of epidemics on scale free networks [5] is also an instance of a binary state dynamics [6].

One issue of interest concerns the importance of the binary assumption: what would happen if opinions were a continuous variable such as the worthiness of a choice (a utility in economics), or some belief about the adjustment of a control parameter? These situations are often encountered in economic and social sciences:

- In the case of technological changes economic agents have to compare the utilities of a new technology with respect to the old one, and for example surveys concerning the adoption of environment friendly practices following the 1992 new agricultural policies [7] showed that agents have uncertainties about the evaluation of the profits when they adopt the new technique and thus partially rely on evaluations made by their "neighbours".
- Some social norms such as how to share the profit of the crop among landlords and tenants [8] do display the kind of clustering that we will further describe.

In the bounded confidence model of continuous opinion dynamics proposed by Deffuant et al. [9], agents can influence each other's opinion provided that the opinions in question are already sufficiently close enough. A tolerance threshold d is defined, such that agents with difference in opinion larger than the threshold can't interact. Several variants of the model have been proposed in [9–11]. In these models, the only restriction on the interaction is the threshold condition and interactions among any pair of agents may occur. The attractors of the dynamics are clusters for which the number increases in steps when the tolerance threshold is decreased.

The dynamics which we will describe here can also be compared to the cultural diffusion model introduced by Axelrod in which agents' culture is represented by strings of integers [12].

The purpose of this paper is to check the role of specific interaction structures on the result of the dynamics. We will investigate a bounded confidence interaction process on scale free networks and compare the obtained dynamics to what was already observed when all interactions are possible and when they occur on square lattices among nearest neighbours.

The paper is organised as follows:

- We first expose the simple case of complete mixing among agents.
- We then check the genericity of the results obtained with the simple model (complete mixing) to other topologies, mostly scale free networks.

We are mainly interested in:

- the clustering process,
- the possible existence of regime transitions according to the value of the threshold of influence d,
- the relative importance of the clustering process with respect to the whole population. Do all or at least most agents participate into this process?

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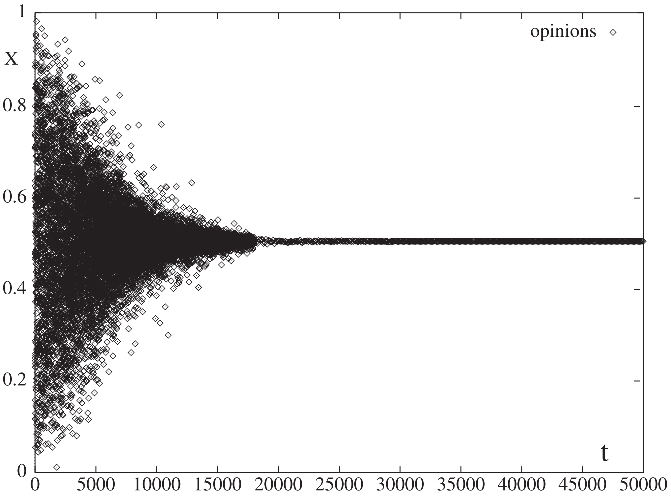

Fig. 1. Time chart of opinions ($d = 0.5 \quad \mu = 0.5 \quad N = 2000$). One time unit corresponds to sampling one pair of agents.

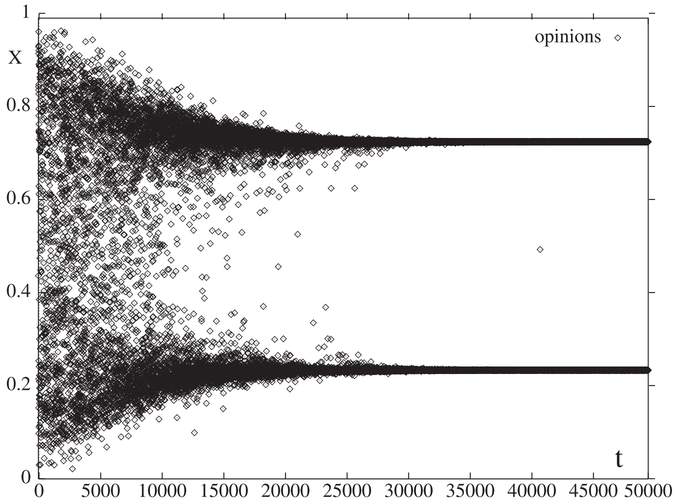

Fig. 2. Time chart of opinions for a lower threshold $d = 0.2$ ($\mu = 0.5 \quad N = 1000$).

2 The basic case: complete mixing

Let us consider a population of N agents i with continuous opinion x_i. We start from an initial distribution of opinions, most often taken uniform on [0,1] in the computer simulations. At each time step any two randomly chosen agents meet: they re-adjust their opinion when their difference in opinion is smaller in magnitude than a threshold d. Suppose that the two agents have opinion x and x'.

Iff $|x - x'| < d$ opinions are adjusted according to:

$$x = x + \mu(x' - x) \tag{1}$$

$$x' = x' + \mu(x - x') \tag{2}$$

where μ is a convergence rate whose value may range from 0 to 0.5.

In the basic model [9], the threshold d is taken as constant in time and across the whole population. Note that we apply here a complete mixing hypothesis plus a random serial iteration mode[1].

For finite thresholds, computer simulations show that the distribution of opinions evolves at large times towards clusters of homogeneous opinions. Such a clustering is a convergence process: all opinions inside a cluster converge to a consensus at infinite time. The dynamics always end up gathering opinions in clusters on the one hand, but also separating the clusters in such a way that agents in different clusters don't exchange anymore.

In the two time charts (Figs. 1 and 2), points correspond to the opinions of those pairs of agents updated at time t, the abscissa of the chart. The distribution of agents is initially rather large and evolves towards clusters of similar opinions at large times.

- For large threshold values ($d > 0.25$) only one cluster is observed at the average initial opinion. Figure 1 represents the time evolution of opinions starting from a uniform distribution of opinions.
- For lower threshold values, several clusters can be observed (see Fig. 2). Consensus is then NOT achieved when thresholds are low enough.

Because opinions have real values, the convergence towards a consensus inside a cluster never actually ends, but the separation between clusters is ensured as soon as no opinion closer than d remains in the neighborhood of a given cluster.

The number of clusters varies as the integer part of $1/2d$: this is to be further referred to as the "1/2d rule" (see Fig. 3[2]).

3 The scale free network topology and opinion updating process

We use a standard construction method, see for example Stauffer[3] and Meyer-Ortmanns [13]:

Starting from a fully connected network of 3 nodes, we add iteratively nodes (in general up to 900 nodes) and connect them to previously created nodes in proportion to their degree. We have chosen to draw two symmetric connections per new added node in order to achieve the same

[1] The "consensus" literature [10] most often uses a parallel iteration mode and supposes that at each time step agents average the opinions of their neighbourhood. The implicit rationale for parallel iteration is that this process models successive meetings among experts.

[2] Notice the continuous transitions in the average number of clusters when d varies. Because of the randomness of the initial distribution and of pair sampling, any prediction on the outcome of dynamics such as the 1/2d rule only becomes true with a probability close to one in the limit of large N.

[3] The model presented here has many similar features to the one studied by Stauffer and Meyer-Ortmanns. But we don't monitor the same quantities which results in rather different perspectives (see later).

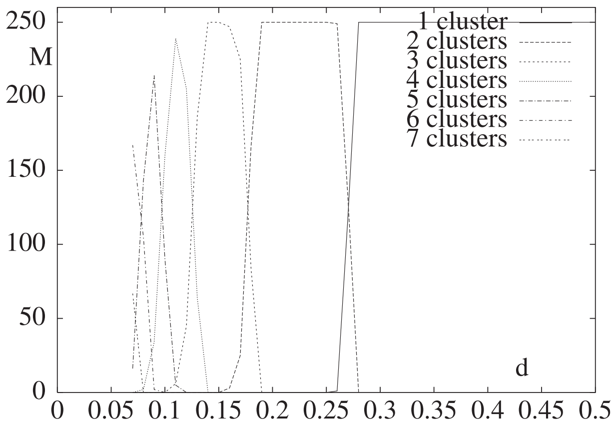

Fig. 3. Statistics of M the number of samples with a given number of clusters as a function of threshold d with 250 samples per d value ($\mu = 0.5, N = 1000$).

average connection degree (4) as in the 30×30 square lattice taken as the reference. But obviously the obtained networks are scale free as shown by Barabasi and Albert [5].

In fact scale free networks [5] display a lot of heterogeneity in node connectivity. In the context of opinion dynamics, well connected nodes might be supposed to be more influential, but not necessarily more easily influenced. At least this is the hypothesis that we choose here. We have then assumed asymmetric updating: a random node is first chosen, and then one of its neighbours. But only the first node in the pair might update his position according to equation (1), not both. As a result, well connected nodes are influenced as often as others, but they influence others in proportion to their connectivity. This particular choice of updating is intermediate between what Stauffer and Meyer-Ortmanns [13] call directed and undirected versions.

4 Clustering and transitions

A simple way to check clustering, and especially its average for many samples, is the dispersion index y proposed by Derrida and Flyvberg [14]. y is the ratio of the sum of the squared cluster sizes s_i^2 to the squared number of agents

$$y = \frac{\sum_{i=1}^{n} s_i^2}{(\sum_{i=1}^{n} s_i)^2}. \tag{3}$$

For m clusters of equal size, one would have $y = 1/m$. The smaller y, the more important is the dispersion in opinions. y can be considered as some average of the inverse number of clusters. In the limit of large N, for uniform initial distributions and slow convergence rates, y is a good indicator of the cluster number.

In the case of full mixing discussed in the previous section, a plot of y as a function of d displays a staircase, in which steps correspond to the discontinuities of the $integer(1/2d)$ function, with y values varying from 0 to 1 (Fig. 4).

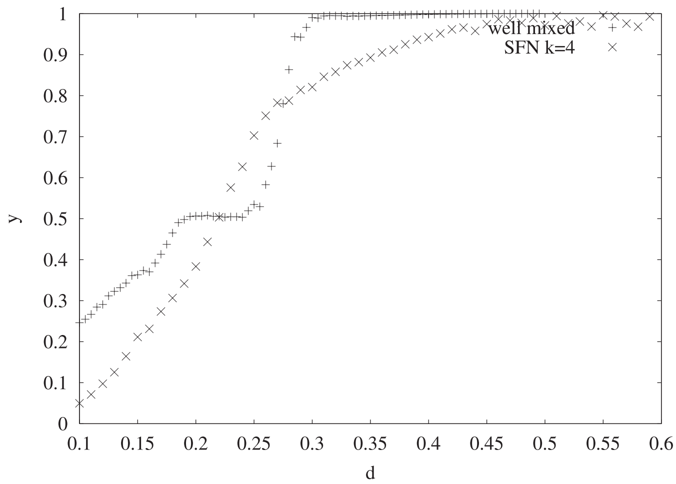

Fig. 4. Average dispersion index y as a function of the tolerance threshold d for well mixed systems ('+') and for scale free networks ('×') with 900 nodes. Each data point is the result of an average over 100 simulations, i.e. 10 initial conditions taken on 10 networks.

When averaging over network topology and initial conditions the step structure (Fig. 4) observed in the case of full mixing seems to be completely blurred for scale free networks: one observes a continuous increase of the Derrida Flyvberg parameter as a function of the tolerance threshold with only a kink in the $d = 0.25, y = 0.7$ region; while three distinct steps at $y = 1$, $y = 0.5$ and $y = 0.33$ are observed in the well mixed case, corresponding to the occurrence of 1, 2 and 3 large clusters respectively.

In fact the blurring of the transition in scale free networksis due to two effects:

- the S curve is the result of averaging over many network topologies and initial conditions.
- The existence of many outlying[4] nodes [15] in scale free networks, which remain out of the clustering process thus decreasing y.

Zooming in one transition region, $0.2 < d < 0.3$, and avoiding averaging give us a better insight (see Fig. 5). When measurements are done on single instances of network topology and initial conditions, one observes in the transition region distinct y values corresponding to either one (larger y values) or two clusters (smaller y values). The proportion of these two y values varies with d, larger y values being more often obtained with larger d values. The continuous variation of the average dispersion index y observed in Figure 4 in this region thus reflects this change in proportion.

[4] During the iterative process of opinion exchange, nodes with few connections have less chances to interact with a neighbour whose opinion is close enough to their own opinion to actually interact. Many of them are not affected by the convergence process and remain outside the distribution of clustered opinions. We call them outlying nodes. Their number decreases with μ the convergence speed parameter. Using physical terms, outlying nodes are "defects" resulting from fast quenching.

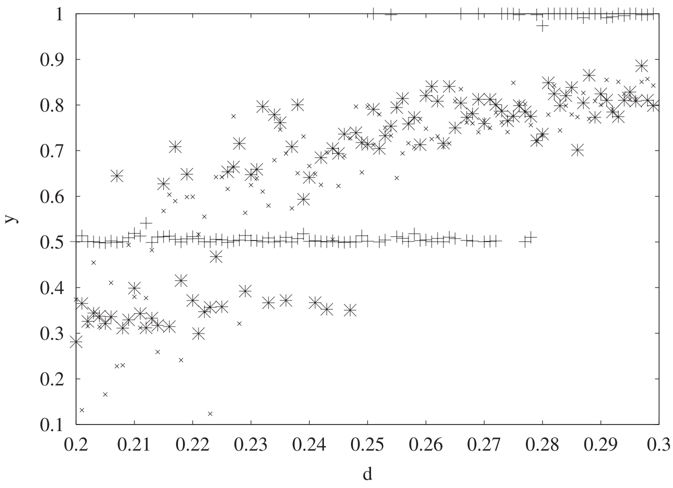

Fig. 5. Variation of the dispersion index y as a function of the tolerance threshold d. Big '+' correspond to the well-mixed case, small '×' to square lattice and big '*' to scale free network with connectivity 4. In contrast with the previous figure, each point is the result of only one sample. The square lattice contains 30×30 nodes and the two other sets contain 900 agents.

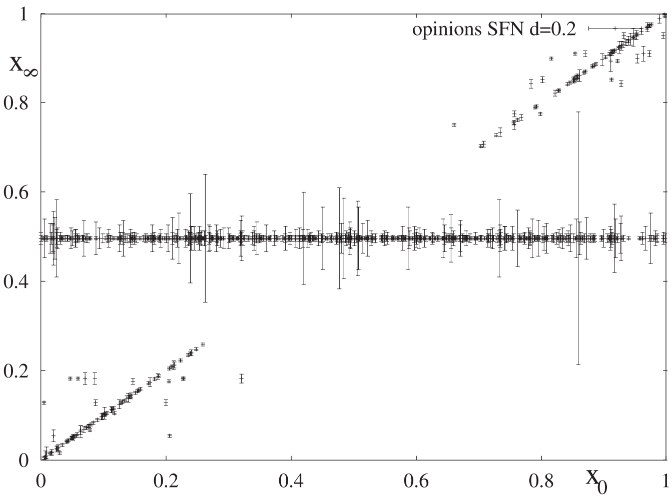

Fig. 6. Final opinions x_∞ versus initial opinions x_0 on a scale free network with average connectivity 4 and tolerance 0.2. Vertical bars give the number of neighbours of each node (the largest correspond to 85).

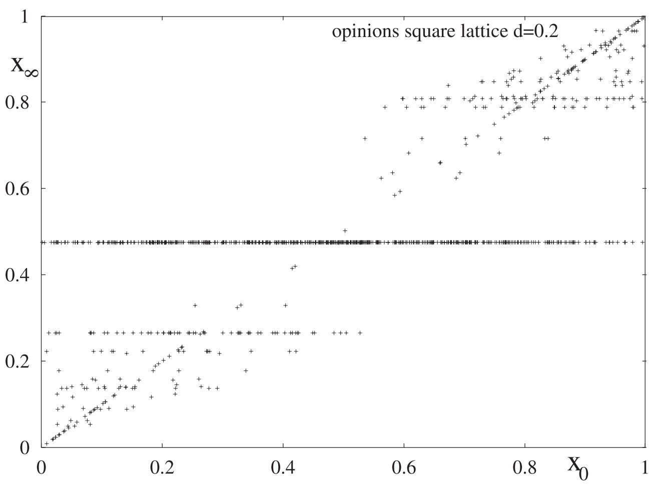

Fig. 7. Final opinions x_∞ versus initial opinions x_0 on a 30×30 square lattice with tolerance 0.2.

For the sake of comparison Figure 5 displays the variations of the dispersion index y with the tolerance threshold d for three different topologies: the standard well-mixed case where any agent might interact with any other one, the square lattice and the scale free network with an average connectivity k equal to 4 ($k = 4$ is also the connectivity of the square lattice).

One observes that in the well mixed case the y values are either 0.5 or 1, with a rather narrow ambiguous region in d. But for scale free networks, y values are smaller, an indication of the existence of many outlying agents whose opinion does not cluster because they are too isolated (also see Figs. 6 and 7). Their distribution is bimodal in a larger ambiguous region. The magnitude and dispersion of y values is similar for the scale free network with connectivity 4 and square lattices. Increasing the average connectivity by a factor 2 (not reported on the figure) brings the scale free network results closer to those of the well-mixed case. Connectivity seems as important as the regularity of connections in determining y.

One of the most important questions in scale free networksis the role of the most connected nodes with respect to the less connected ones. In the context of opinion dynamics, we might want to figure out whether they are more influential, or eventually more influenced? One answer is provided by checking how far their opinion is changed by the clustering process. Figure 6 is a plot of the opinions of agents after convergence of the clustering process as a function of their initial opinion. Node connectivity is indicated by the size of the vertical bars. The importance of clustering is indicated by the density of points on horizontal lines while outlying agents are located on the first bisector.

Most of the well connected nodes belong to the horizontal cluster at $x_\infty = 0.5$. Most are away from the first bisector, which imply that they have been influenced during the clustering process.

The first bisector is composed of less connected nodes, whose initial and final opinions are more than $d = 0.2$ away from the cluster. These nodes have not changed their opinion. In scale free networks, static isolation (due to lower connectivity) often results in being kept out of the clustering process and remaining outlying. The effect is systematically observed for all tolerance thresholds less than 0.5. The outlying nodes' number explains why the highest values of y are lower than 1 in Figure 5: only one central cluster is present, but it only contains a fraction of the nodes.

For the same d values, well mixed systems display horizontal clusters in this $[x_0, x_\infty]$ representation but few outlying agents (not represented here).

Stauffer et al. [13] have done extended statistics on the total number of different opinions after convergence in scale free networks. Since the number of outlying agents is much bigger than the number of big clusters, their figures give a very good characterisation of the number of outlying nodes.

For the sake of comparison we give the equivalent display for square lattices (Fig. 7). The results are pretty similar to those obtained with scale free networks. The less populated horizontal lines around $x_\infty = 0.2$ and $x_\infty = 0.8$ correspond to small homogeneous "islands" on the lattice while the big $x_\infty = 0.5$ region percolates as checked on computer displays [9] (not represented here).

5 Conclusions

In conclusion, restricting influence by a network topology does not drastically change the behaviour of these models of social influence as compared to the well mixed case. To summarise some of the resemblances and differences:

– One does observe clustering effects, and the number of observed main clusters does not differ much from what is observed for equivalent tolerance thresholds in the well mixed case[5].
– Stairs of y, the dispersion index, do appear: at least when measured without averaging on single instances of networks and initial conditions. But y values are decreased by a larger proportion of outlying agents and the transition regions in tolerance are larger.
– Well connected nodes are influenced by other nodes and are themselves influential. Most of them belong to the big cluster(s) after the convergence process.
– Larger connectivities bring the scale free networks dynamic behaviour closer to well mixed systems.

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[5] We have only been discussing clusters in terms of opinions, not in terms of connections across the network. For small d values, clustering in opinion might structure the network in smaller connected regions with clustered opinions. One can expect the number of such non-interacting regions to be larger than the number of clusters (as observed on square lattices [9]).